$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$ $E_b = \sigma T^4$

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$e^{i\pi} = -1$ $\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\frac{y u_\tau}{\nu} + C$

USER GUIDE

Conical Shock Web Application

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21 September 2020

Atkinson Science welcomes your comments on this User Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

Contents

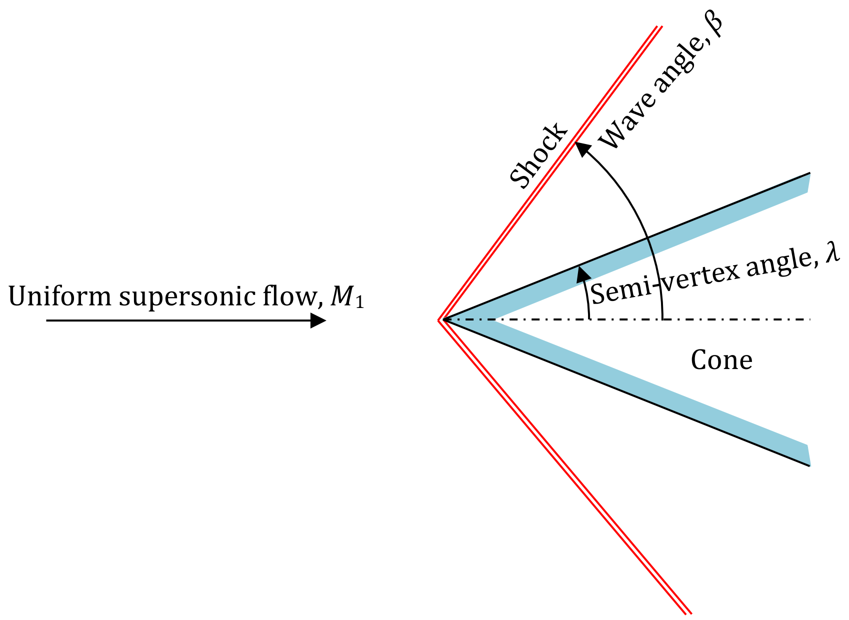
Figures

1 Introduction

You can find the Atkinson Science Conical Shock web application at the web address https://atkinsonscience.co.uk/WebApps/Aerospace/ConicalShock.aspx. The application calculates the change in properties across the conical shock created around a cone in a uniform supersonic flow. It was developed for use with the International Standard Atmosphere, Ref. [1], for which the ratio of specific heats γ is defined to be 1.4. The application was created in Microsoft Visual Studio 2017. The assumptions and equations on which the application is based are set out Ref. [2], which can be downloaded from the Atkinson Science web site:
https://atkinsonscience.co.uk/PDFs/WebApps/Conical%20Shock%20Theory%20Guide.pdf.

A conical shock is created when a sharp cone is placed in a uniform supersonic flow with the axis of the cone parallel to the flow. We shall assume that the cone extends to infinity in the downstream direction (the cone is semi-infinite). The conical shock is formed at the vertex of the cone, as shown in Figure 1. The change in properties across the shock can be assumed to be the same as for an oblique shock formed at the vertex of a sharp wedge. However, the wave angle β of the conical shock will be less than that of the oblique shock, assuming the semi-vertex angle λ of the cone and the deflection angle of the wedge are the same.

Figure 1 Conical shock



2 Shock properties

The conical shock formed around a cone has the characteristics of an oblique shock and can be calculated from oblique shock relations if the wave angle β of the shock is known. However, the flow downstream differs from that of an oblique shock. After traversing the conical shock, the flow streamlines curve until they become parallel with the surface of the cone at infinity. In contrast, the streamlines behind an oblique shock become parallel to the surface of the wedge immediately. Since the cone extends to infinity the idea that flow properties may vary along the surface of the cone is not meaningful. In fact, experiments show that the flow properties are constant along the surface of the cone.

Usually, it is a weak oblique shock that is formed around the cone, rather than a strong oblique shock. However, if the pressure at the base of the cone can be increased, then it is possible to create a conical shock with the characteristics of a strong oblique shock (just as when the pressure at the base of a wedge is increased). Both types of shock are attached to the vertex of the cone, but the changes in flow properties across the strong shock are more severe. The web application computes only the weak shock solution.

Figure 2 applies to the weak shock solution and shows how the wave angle β varies with the semi-vertex angle λ and the upstream Mach number M_1. The ratio of the specific heats γ is assumed to be 1.4. For each Mach number M_1 there is a highest possible semi-vertex angle λ_{max} and a highest possible wave angle β_{max}.

If the semi-vertex angle exceeds the λ_{max} value for the upstream Mach number, then the shock becomes detached, as shown in Figure 3. The web application does not deal with the solution of the detached shock. The highest semi-vertex angle λ_{max} at which a conical shock can be formed increases with M_1, as can be seen from Figure 2. The web application can determine whether the user's input would produce a detached shock. If this is the case, then the application issues a warning message and advises the user to either reduce the semi-vertex angle or increase the upstream Mach number.

Figure 2 Wave angle against semi-vertex angle for different Mach number values

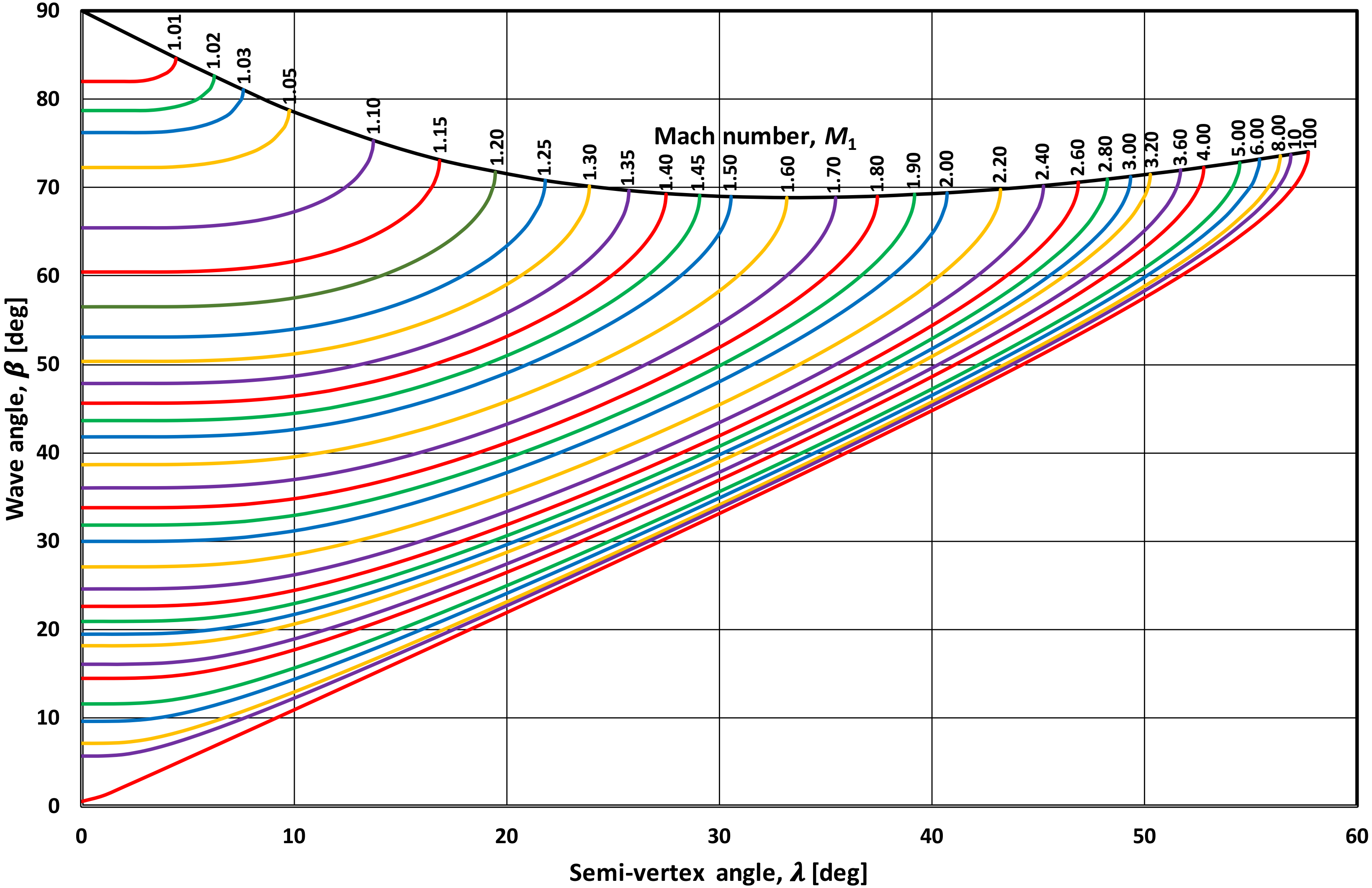
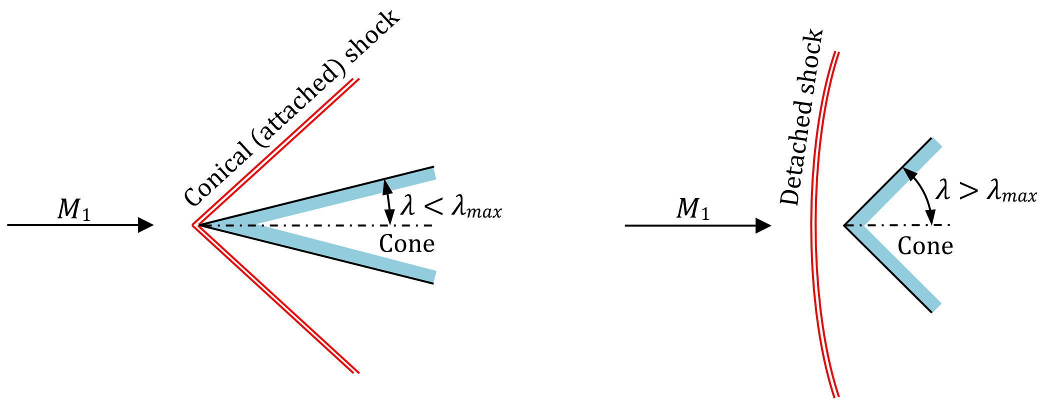


Figure 3 Conical and detached shocks



3 Solution of the flow

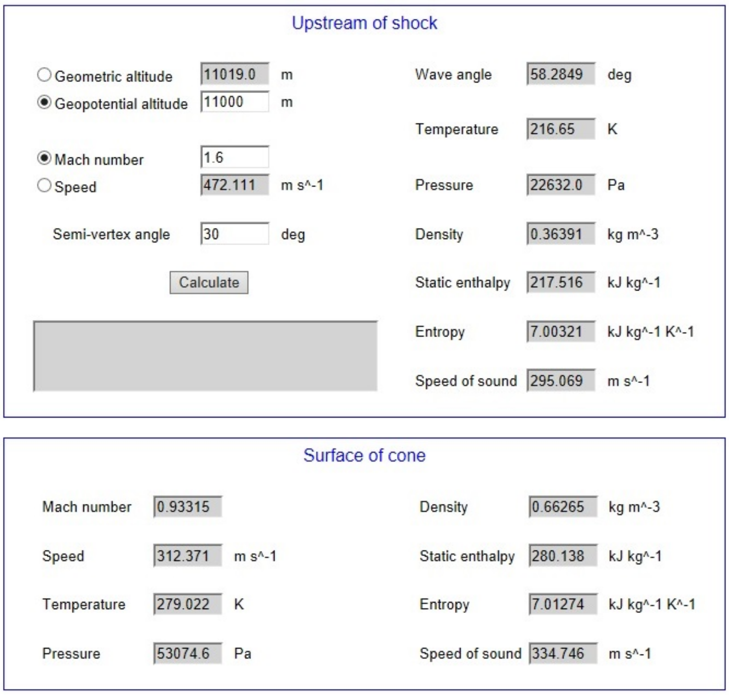
A method of solving the flow behind a conical shock has been published by Taylor and Maccoll, Ref. [3]. They assume that the flow behind the shock is irrotational. This assumption leads a considerable simplification of the equations describing the flow behind the shock. The method yields the wave angle of the shock and the properties of the flow (speed, temperature, pressure, etc.) at the surface of the cone. A description of the solution method can be found in text books on compressible flow, such as Ref. [4]. Also, along with this user guide, there is a theory guide, Ref. [2], that can be downloaded from the Atkinson Science web site.

The web application is intended for use with the International Standard Atmosphere, Ref. [1]. It is then only necessary to specify the geometric or geopotential altitude to obtain the temperature, pressure and other thermodynamic properties of the air upstream of the shock. To complete the input, the Mach number or speed of the flow and the semi-vertex angle of the cone must be specified. The ratio of the specific heats γ of the International Standard Atmosphere is defined to be 1.4.

4 User Interface

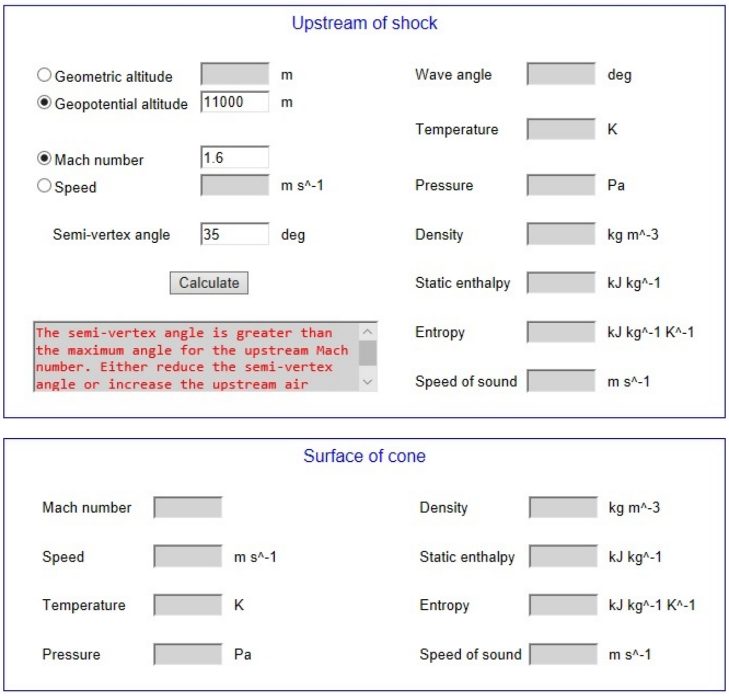
Figure 4 shows the user interface of the conical shock web application. The user must enter either the geometric altitude or the geopotential altitude in the International Standard Atmosphere, the Mach number or speed of the supersonic flow, and the semi-vertex angle of the cone. The user then clicks the Calculate button to calculate the properties of the atmosphere upstream of the shock, the wave angle of the shock, and the properties of the flow at the surface of the cone. In Figure 4, the user has entered a geopotential altitude of 11,000 m, a Mach number M_1 of 1.6, and a semi-vertex angle λ of 30°. The calculated wave angle β is 58.2849°, and this value is consistent with the plot of wave angle β against semi-vertex angle λ for different upstream Mach numbers M_1 in Figure 2. The shock wave gives rise to an increase in temperature, pressure and density, and the flow at the surface of the cone is subsonic.

Figure 4 User interface



In Figure 5, the user has increased the semi-vertex angle λ to 35°. This angle exceeds the highest semi-vertex angle for the upstream Mach number and would produce a detached shock, which cannot be calculated by the web application. The application issues a warning message and advices the user to either reduce the wave angle or increase the Mach number.

Figure 5 Warning message



5 References

1. *International Standard Atmosphere*, ISO 2533:1975, International Standards Organisation, 1975.
2. K. N. Atkinson, Conical Shock Web Application, Theory Guide, Atkinson Science Limited, 21 September 2020 (download from https://atkinsonscience.co.uk).
3. G. I. Taylor and J. W. Maccoll, The Air Pressure on a Cone Moving at High Speeds – I, Proc. Royal Society, Series A, Vol. 139, 1933, pp. 278-311.
4. J. D. Anderson, *Modern Compressible Flow with Historical Perspective*, 3rd Ed., McGraw-Hill, 2004.